Melting of a Horizontal Substrate Placed Under a Heavier and Miscible Pool

K. Taghavi*
University of Kentucky, Lexington, Kentucky

The melting of a horizontal substrate under a heavier miscible pool is studied. This phenomenon has applications in nuclear reactor safety where, in the event of a hypothetical core disruptive accident, molten UO_2 may come in contact with and melt the core structural material. Knowing that the growth and geometry of the melt layer is governed by Taylor instability, a model based on the hydrodynamics and heat transfer of the melt layer is used to obtain a relation for the melting rate and heat flux. The analysis includes the radial flow of the melt, is not restricted to uniform melt layers, and considers the uneven shape of the melting solid. A heat flux correlation that depends on the relevant melt properties and reveals the dependence of the heat flux on the pool-to-melt density ratio and the temperature difference across the melt layer is obtained. The results are compared with the available experimental data.

Nomenclature

c_p = specific heat of the melt
$c_{p,s}$ = specific heat of the solid substrate
$c_{p,p}$ = specific heat of the pool
D = coefficient of diffusion of the melt into the pool
g = gravitational acceleration
h_{sf} = latent heat of fusion
h'_{sf} = latent heat of fusion corrected for subcooling
Ja = Jakob number, $c_p(T_s - T_m)/h_{sf}$
k = thermal conductivity of the melt
k_p = thermal conductivity of the pool
$\dot{m}''$ = melting flux
$\dot{m}''_D$ = mass flux due to diffusion
$m_{A,s}$ = mass fraction of the melt material at the pool-melt interface
$m_{A,p}$ = mass fraction of the melt material in the pool
n = an integer, 1 or 2, representing different types of boundary conditions
p = pressure
Pr = Prandtl number
q'' = heat flux across the melt layer
r = radial coordinate
r_j = radius of the jet
r_o = unit cell radius
R = pool-to-melt density ratio, ρ_p/ρ
t = time
T = temperature
T_m = melting temperature
T_p = pool temperature
T_s = temperature at the interface of melt and pool
u = melt velocity in the r direction
u_b = mean melt velocity in the r direction
u_j = jet velocity in the z direction
z = coordinate normal to the melting substrate
α_p = thermal diffusivity of the pool
β = power in the relation $9'' \propto (R-1)\beta$
δ = melt layer thickness
ΔT = temperature difference across the melt layer, $T_s - T_m$
ΔT_{sub} = subcooling of the melt substrate
$\Delta\rho$ = difference between the pool and melt densitites, $\rho_p - \rho$
η = parameter denoting the shape of the solid substrate
λ = Taylor wavelength
μ = viscosity
ν = kinematic viscosity
ρ = density of the melt
ρ_p = density of the pool
ω = penetration velocity, melting velocity

Introduction

THE melting of a horizontal substrate under a heavier pool has been the subject of several studies. The situation arises in a hypothetical nuclear reactor core disruptive accident, where a layer of liquid UO_2 may form over a horizontal surface made of steel, concrete, or a sacrificial material such as MgO. As a result, a molten layer, which is generally lighter than the liquid UO_2, will be formed. It has been shown by Taylor[1] and later by Bellmann and Pennington[2] that such a system is hydrodynamically unstable and that its instability and growth is subject to Taylor instability.

Zuber[3] was the first to apply Taylor instability to a phase change system and model pool boiling over a horizontal surface. Later, Berenson[4] analyzed film boiling by assuming an average shape for the film geometry and that the spacing between the vapor jets was equal to the Taylor wavelength. In an attempt to study the melting problem, Dhir et al.[5] examined the sublimation of dry ice under a pool of water. They followed Berenson's technique and obtained an expression for the heat transfer coefficient. Subsequently, Taghavi et al.[6] studied a melting system where the ratio of the pool-to-melt density R was about 1.1. They obtained a melt layer geometry and a relation for the heat transfer coefficient by balancing the buoyancy and surface tension forces. One important observation by Taghavi et al.[6] was that the pool became thermally stratified and that a better agreement between theory and experiment would be achieved if the heat transfer coefficient was based on the temperature difference

Presented as Paper 86-HT-11 at the AIAA/ASME 4th Thermophysics and Heat Transfer Conference, Boston, MA, June 2–4, 1986; received Sept. 2, 1986; revision received Jan. 20, 1987.
*Department of Mechanical Engineering.

across the melt layer rather than the temperature difference between the pool and the melt.

Farhadieh and Baker[7] were the first to study a melting system where the melt and the pool were miscible. They conducted experiments by placing layers of Carbowax under aqueous solutions of KI and NaBr. They too concluded that the melting process was governed by Taylor instability and that it was sensitive to the density ratio R and to the temperature difference between the pool and the melt. They also observed that the pool became thermally stratified after it was poured over the melt material. Farhadieh and Baker, however, did not attempt to model the melting phenomenon.

Recently Catton et al.[8] studied heat transfer from a heated pool to a melting miscible substrate. Based on an analysis similar to that by Berenson,[4] they obtained an expression for the heat transfer coefficient, which upon adjustment of its numerical constant yielded a satisfactory match between their theory and experimental data. However, it did not agree as favorably with the data obtained by Farhadieh and Baker.[7] They also observed that the melting rate was highly sensitive to the density ratio, but reported that it was not dependent on the pool-melt temperature difference. One important parameter that is notably missing from the heat transfer coefficient correlation obtained by Catton et al.[8] is the thermal conductivity of the melt layer. This is somewhat unusual and different from most of the other studies of similar phenomena, e.g., Berenson.[4] In a Berenson-type modeling, the thermal conductivity of the melt seems to be essential, since the heat has to be transferred across the melt layer in order to melt the solid.

Fang et al.[9] studied this melting problem experimentally. They too observed that the melting rate was dependent on the density ratio and, as Catton et al.[8] had observed, independent of the pool temperature. However, neither Catton et al.[8] nor Fang et al.[9] reported any thermal stratification in the pool.

Very recently, Epstein and Grolmes[10] studied the natural convection characteristics of pool penetration into a melting and miscible solid. Their study was experimental in nature and included experiments with several combinations of the pool and melt materials. Some of the combinations were identical to those used by Farhadieh and Baker[7] and Catton et al.[8] They discovered that the latent heat of melting for Carbowax was strongly dependent on the solid temperature and that this dependence was overlooked by Farhadieh and Baker.[7] Epstein and Grolmes[10] also observed that their experimenal results were up to five times smaller than those obtained by Catton et al.[8] for the same combination of materials. They did not offer an explanation for the latter difference. The two major conclusions by Epstein and Grolmes are that the melting rate is linerly proportional to the temperature difference and proportional to the density difference to approximately the ⅓ power.

Epstein and Grolmes[10] suggested that the role of the pool natural convection on the melting process is both complicated and important. They observed that little mixing occurs adjacent to the melting surface and that the convection cells exist somewhat above the melt layer. Naturally, for a complete modeling of this complex phenomenon, a treatment that couples the natural convection in the pool with the melting process is necessary.

Obviously, there lacks a modeling effort that would predict the dependence of the heat flux on the temperature and density difference that matches with the majority of the experimental data. This paper focuses on the nature of the melting process and would not attempt to consider the natural convection in the pool. Accordingly, the analysis is performed as if the pool-melt interface temperature T_s is known. This would decouple the pool's natural convection and the melting process. As a result, this work cannot completely predict the melting rate; rather, it will reveal how the melting rate depends on the melt properties. In this paper, an attempt will be made to model the melting phenomenon based on physical understanding of the problem without any influence from the experimental data. Then the results will be compared with the available experimental data.

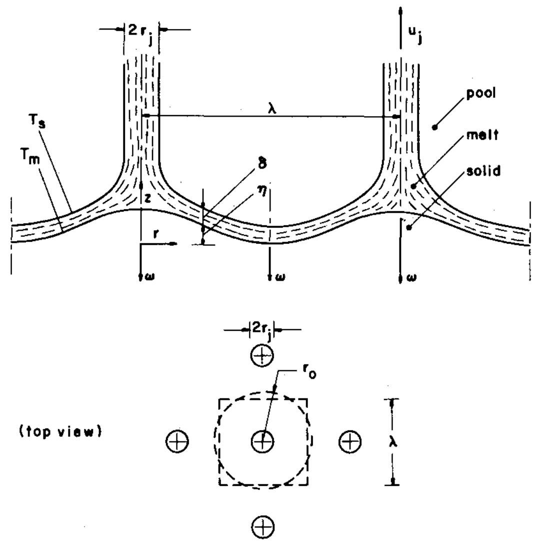

Fig. 1 Physical model for the melting of a horizontal substrate placed under a heavier and miscible pool.

Modeling

As the solid substrate is placed under the heated pool, heat transfer from the pool causes the substrate to melt. As a result, a thin layer of the melt liquid forms between the melting solid and the hot pool. Since the pool liquid is heavier than the melt, their interface becomes unstable and the melt is removed in the shape of jets (see Fig. 1). Originally, when the melting surface is flat, there is no clear driving force for the movement of the melt. But once the jet sites are chosen under the influence of Taylor instability, the melt starts to flow radially toward the jet sites. As the melt flows toward the jets, the radius decreases and so does the area available for the flow of melt. To accommodate the flow of melt, the melt layer thickness has to increase, which is turn would introduce the buoyancy force necessary to drive the melt flow. It may be argued that the process of melting a horizontal flat surface under a heavier pool is a self-starting one.

A higher melting rate would require a thicker melt layer to accommodate the removal of the melt. On the other hand, the thicker the melt layer, the larger the thermal resistance and the smaller the heat flux into the melting surface. The balance of these two opposing effects would dictate the melt layer geometry. The melting rate in the areas near and under the jet columns, where the melt layer is thicker, is smaller than the regions farthest from the jets. This would cause uneaven melting of the solid, which in time would change the shape of the melting solid. A steady-state shape for the melting surface would be achieved when the penetration velocity becomes independent of the radial position.

Figure 1 shows the physical model used for analyzing the melting process. The following assumptions are made:

1) The solid is at its melting temperature. Therefore any heat transfer to the solid will cause melting of the solid. However, the analysis will be valid for subcooled substrates as long as the conduction of heat into the solid is faster than the melting rate. For such a situation, the latent heat of fusion can simply be corrected for the substrate temperature being lower than the melting temperature as

$$h'_{sf} = h_{sf} + c_{p,s}\Delta T_{\text{sub}} \tag{1}$$

where ΔT_{sub} is the subcooling of the substrate.

2) The locations for the jet sites are determined and governed by Taylor instability. Therefore, the jets are apart by a Taylor wavelength λ.

3) The pool is separated from the substrate by a thin layer of melt. This is assumed in spite of the fact that the melt and the pool are miscible liquids. In other words, it is assumed that the rate of diffusion of melt into pool from the melt film is much smaller than the rate of melting. The validity of this assumption will be checked in Appendix A.

4) The pool-melt interface temperature T_s is assumed to be uniform and not changing with radial position.

5) The melt layer thickness is small enough ($\delta/\lambda \ll 1$) so that the inertia forces and the convective terms in the energy equation are negligible. As a result the temperature profile across the melt layer is assumed to be linear. The validity of this assumption will be checked in Appendix B.

6) The peaks and valleys in the melting solid are small enough [$\eta(0)/\lambda \ll 1$] so that the melt flow could be approximated as a radial flow. The validity of this assumption will be checked in Appendix B.

7) The melt flow in the jets is due to a balance between the buoyancy and viscous forces, i.e., the inertia fores are negligible. This assumption is reasonable for jets with small radius and velocity.

8) The shape of the melting surface has reached a steady state. In other words, the penetration velocity ω is not a function of the radial position r.

9) The heat transfer mechanism in the pool will not be considered here. However, this does not mean that such a mechansim has been neglected. The addressing of the heat transfer mechanism in the pool is avoided by correlating the results in terms of the temperature difference across the melt layer rather than the difference between the pool average and melt temperature

Taylor Wavelength

It is assumed that the removal of the melt is governed by Taylor instability, and therefore the melt jets are spaced apart by a Taylor wavelength. It was shown by Taghavi and Dhir[11] that, for a system of two miscible and infinitely thick layers with equal kinematic viscousity, the Taylor wavelength is calculated as

$$\lambda = 4\pi\left(\frac{R+1}{R-1}\right)^{1/3}(\nu^2/g)^{1/3} \tag{2}$$

Here, the model is based on a unit cell shown in Fig. 1. The radius of the cell is obtained by preserving the melting area that contributes to a single jet. Therefore

$$\pi r_o^2 = \lambda^2 \tag{3}$$

Momentum Equation

The momentum equation in the r direction for the melt layer may be written as

$$\mu\frac{\mathrm{d}^2u}{\mathrm{d}z^2} = \frac{\mathrm{d}p}{\mathrm{d}r} = -\Delta\rho g\frac{\mathrm{d}}{\mathrm{d}r}(\eta+\delta/2) \tag{4}$$

By writing Eq. (4), it is assumed that the inertia terms in the melt layer are negligible and that the melt flow is driven by the buoyancy force. The boundary conditions for the velocity in the melt layer are

$$u = 0 \quad \text{at} \quad z = 0 \tag{5}$$

and

$$\frac{\mathrm{d}u}{\mathrm{d}z} = 0 \quad \text{at} \quad z = \frac{\delta}{n} \tag{6}$$

where $n=1$ represents the case of no shear stress and $n=2$ represents the case of zero velocity at the melt-pool interface. Equation (4) may be integrated twice while using the boundary conditions (5) and (6) to obtain the velocity profile $u(z)$. The mean velocity may then be calculated by integrating $u(z)$:

$$u_b = \left(\frac{3-n}{6n}\right)\left(\frac{\Delta\rho g\delta^2}{\mu}\right)\frac{\mathrm{d}}{\mathrm{d}r}(\eta+\delta/2) \tag{7}$$

Mass Balance

The mean velocity may also be related to the penetration velocity by applying a mass balance on a melt element dr:

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\delta u_b) = r\omega \tag{8}$$

The integration of Eq. (8) from r_o to r yields

$$u_b = -\frac{r_o^2 - r^2}{2r\delta}\omega \tag{9}$$

Energy Balance

An energy balance may be applied to the melt element dr in order to relate the melting rate to the heat flux across the melt layer:

$$\frac{k\Delta T}{\delta}\mathrm{d}r = \rho \mathrm{d}r\omega h_{sf} \tag{10}$$

or

$$\delta = \frac{k\Delta T}{\rho\omega h_{sf}} \tag{11}$$

An important conclusion may be made from Eq. (11): In a steady-state situation where the shape of the melting solid is not changing with time and therefore the penetration velocity ω is constant and is not varying with r, the film thickness δ has to be uniform. The mean velocity u_b may be eleminated between Eqs. (7) and (9) and δ may be replaced from Eq. (11) to obtain a single equation in η:

$$\frac{\mathrm{d}\eta}{\mathrm{d}r} = -\left(\frac{3n}{3-n}\right)\left[\frac{\mu\omega^4\rho^3h_{sf}^3}{\Delta\rho g k^3\Delta T}\right]\left[\frac{r_o^2-r^2}{r}\right] \tag{12}$$

Equation (12) may be integrated once with an arbitrary boundary condition of $\eta=0$ at $r=r_o$ to obtain

$$\eta = \left(\frac{3n}{3-n}\right)\left[\frac{\mu\omega^4\rho^3h_{sf}^3r_o^2}{\Delta\rho g k^3\Delta T}\right][r^2/2r_o^2 - \ln(r/r_o) - 1/2] \quad \text{for } r_j \le r \le r_o \tag{13}$$

Equation (13) is obviously not valid as $r\to 0$ since $\eta\to\infty$. As a result, in conformance with assumption 6 that led to these analyses, the validity of Eq. (13) is limited to $r_j \le r \le r_o$.

Jet Radius

The criterion used here is that the melt velocity does not change while leaving the melt layer and entering the jet column, i.e.,

$$u_j = -u_b(r_j) \tag{14}$$

Equation (14) and mass continuity dictates that the area available for the melt flow should remain unchanged too.

$$\pi r_j^2 = 2\pi r_j\delta \tag{15}$$

That is,

$$r_j = 2\delta \tag{16}$$

Jet Velocity

The velocity of the jet is due to the balance between the buoyancy forces and the viscous forces. Therefore, the momentum equation for a circular pipe may be solved to calculate the jet velocity as

$$u_j = -\frac{r_j^2}{8\mu}\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{r_j^2 \Delta\rho g}{8\mu} \tag{17}$$

Equation (14) may now be rewritten using Eqs. (17), (16), and (9) and solved for ω:

$$\omega = \frac{2g\delta^4}{\nu(r_o^2 - 4\delta^2)}(R-1) \tag{18}$$

Assuming that $4\delta^2/r_o^2 = r_j^2/r_o^2 \ll 1$, Eq. (18) may be approximated as

$$\omega = \frac{2g\delta^4}{\nu r_o^2}(R-1) \tag{19}$$

The melt layer thickness δ in Eq. (19) may be replaced from Eq. (11) to obtain ω:

$$\omega = \left[\frac{2g(R-1)k^4\Delta T^4}{\nu r_o^2 \rho^4 h_{sf}^4}\right]^{1/5} \tag{20}$$

Knowing the penetration velocity ω, the heat flux may now be written as

$$q'' = \rho h_{sf}\omega = \left[\frac{2g(R-1)k^4\Delta T^4 \rho h_{sf}}{\nu r_o^2}\right]^{1/5} \tag{21}$$

But r_o, the unit cell radius, may be replaced from Eqs. (3) and (2) to obtain the following:

$$q'' = \left[\frac{g(R-1)k^4\Delta T^4 \rho h_{sf}}{8_{\pi\nu}[(R+1)/(R-1)]^{2/3}(\nu^2/g)^{2/3}}\right]^{1/5} \tag{22}$$

Equation (22) may be rearranged in order to be written in terms of the familiar and relevant dimensionless parameters as

$$\frac{q''(\nu^2/g)^{1/3}}{\mu h_{sf}} = \left(\frac{R+1}{8\pi}\right)^{1/5}\left(\frac{R-1}{R+1}\right)^{1/3}\left(\frac{Ja}{Pr}\right)^{4/5} \tag{23}$$

Results and Discussion

A relation based solely on the phenomenological modeling of this melting process has been obtained. It relates the melting rate to a number of parameters that were expected to be influential. These are the thermophysical properties of the melt layer (including the thermal conductivity, viscosity, and latent heat of fusion), the pool-to-melt density ratio, and the temperature difference across the melt layer.

Equation (23) shows a 4/5 power dependence of the heat flux on the temperature difference. This is different than the usual 3/4 power obtained in the studies of similar phenomena.[4-6] This difference is the direct result of the coupling of the jet and the melt layer in the analysis.

It will be interesting to compare the present results to the data available in the literature. Figure 2 examines the temperature difference dependence of the present result by comparing it with those data obtained by Farhadieh and Baker,[7] Catton et al.,[8] and Epstein and Grolmes.[11] The absolute agreement between the present theory and the data is at best marginal. The present model suggests a $\Delta T^{4/5}$ relationship, which is in excellent agreement with the data obtained by Epstein and Grolmes.[10]

Figure 2 reveals an interesting observation regarding the dependence of the heat flux on the temperature difference.

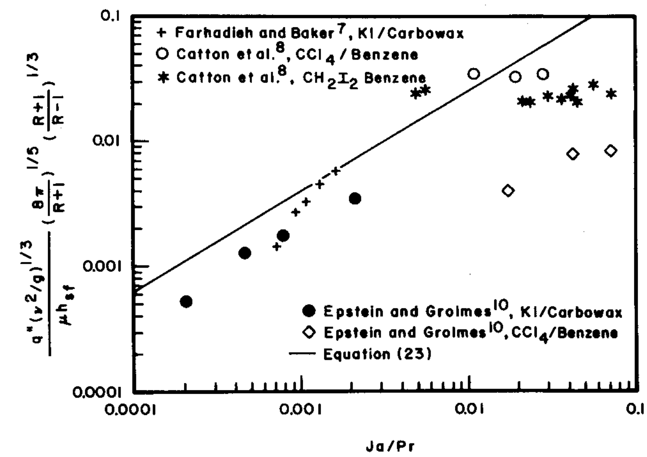

Fig. 2 Dependence of heat flux on temperature difference across the melt layer and comparison with the available data.

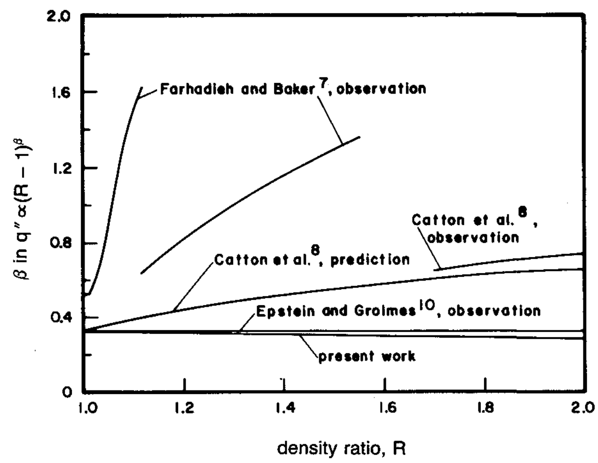

Fig. 3 Dependence of heat flux on density ratio according to the present study and the available data.

The temperature difference dependence of the data by Farhadieh and Baker[7] is stronger than that of the present model. However, when the data by Epstein and Grolmes[10] for the same combination of materials is considered, the present model matches the data satisfactorily. It should be noted again that Epstein and Grolmes[10] revealed a mistake in the work of Farhadieh and Baker[7] in interpreting the thermophysical properties of Carbowax. The same situation is present for the data of Catton et al.[8] They obseved that the heat flux almost independent of the temperature difference between the pool and the melting surface. The present model, on the other hand, indicates that the heat flux is proportional to the temperature difference to the 4/5 power. Again regarding the dependence of the heat flux on the temperature difference, the present model compares favorably with the data of Epstein and Grolmes[10] where they use the same combination of materials as Catton et al.[8]

The two most important parameters involved in this process are the temperature difference and the pool-to-melt temperature ratio. The present model suggests that the heat flux is proportional to the temperature difference to the 4/5 power. This compares well with the data obtained by Epstein and Grolmes.[10] The dependency of the heat flux on the density ratio, however, is more controversial, since different sources claim different trends. Figure 3 plots the behavior of the heat flux vs the density ratio according to the present study and several other studies. The work by Epstein and Grolmes[10]

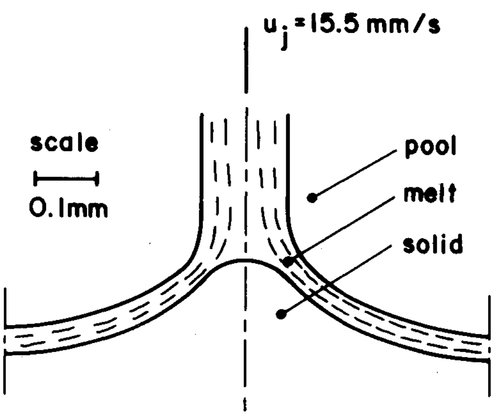

Fig. 4 Geometry of the melt layer for benzene melting under CH_2I_2 when $\Delta T = 13.6$ K and $R = 3.28$. The combination of the pool and the melt materials is adopted from Catton et al.[8]

mostly claims $q'' \sim (R-1)^{1/3}$. The present study reveals a 1/3 power for the density ratios close to 1, and decreasing to 0.31 for the density ratio of 1.4. Therefore, an excellent agreement is observed between the present model and the data of Epstein and Grolmes.[10]

The density ratio power dependence observed by Farhadieh and Baker[7] ranges from 1/2–5/3. This dependency should probably be discounted in light of the discovery by Epstein and Grolmes[10] that the thermophysical properties of Carbowax have not been used properly. Catton et al.[8] show a power dependency of 1/3 for $R = 1$, increasing to 0.5 for $R = 1.4$. The latter is 50% higher than that observed by Epstein and Grolmes[10] for the same combination of materials.

The shape of the melting solid is expressed by Eq. (13) and should be calculated individually for each data point. Unfortunately, there are no experimental data available on the shape of the melting solid for comparison. For the purpose of illustration, Fig. 4 is depicting, to the scale, the shape of the melting solid for one data point obtained by Catton et al.[8] In this figure, the melt layer film thickness and the jet dimensions are also shown to the scale. Figure 4 may be used to estimate a surface roughness of $\eta(0)/\lambda \simeq 0.25$.

Summary and Conclusions

In this paper, a phenomenological model representing the melting of a horizontal surface under a heavier miscible pool is proposed. The modeling effort was focused on the melting process, while the natural convection in the pool was not considered. Therefore, the present model is not expected to predict the quantitative values of the heat transfer, but rather to reveal the dependence of the melting rate on the pool-to-melt density ratio and the temperature difference between the pool and the melt surface. This model has the following features:

1) It includes the effect of the radial flow in the melt layer.

2) It assumes a variable melt layer thickness, although the melt layer is later found to be uniform.

3) It considers the uneven melting of the substrate and offers a relation for the shape of the melting solid.

4) The theoretical result for the dimensionless heat transfer includes all of the important melt properties, e.g., the melt thermal conductivity and the latent heat of fusion.

5) The result does not include any thermal properties of the pool liquid other than the pool density, which is in the buoyancy term. This is due to not considering the heat transfer mechanism in the pool. Actually, irrespective of the heat transfer mode in the pool, there will be some temperature drop in the pool that would depend on the pool properties as well as the melt properties and has to be included in the final presentation of a dimensionless heat flux.

6) The modeling treats the mechanism of the melt flow in the jets and allows for the infleunce of the jet hydrodynamics on the melt layer film thickness and heat transfer.

The following conclusions may be made:

1) A heat flux relation is obtained as

$$\frac{q''(\nu^2/g)^{1/3}}{\mu h_{sf}} = \left(\frac{R+1}{8\pi}\right)^{1/5}\left(\frac{R-1}{R+1}\right)^{1/3}\left(\frac{Ja}{Pr}\right)^{4/5} \tag{24}$$

This equation relates the dimensionless heat flux to Ja/Pr and the density ratio R. The dependence on Ja is found to be of 4/5 power, which is different from the traditional 3/4 power due to the unique characteristics of the process. This $\Delta T^{4/5}$ dependence agrees favorably with the data obtained by Farhadieh and Baker[7] and Epstein and Grolmes.[10]

2) The heat flux is sensitive to the pool-to-melt density ratio. The analytical model indicates that

$$q'' \propto (R-1)^{1/3} \quad \text{for } R \simeq 1 \tag{25}$$

$$q'' \propto (R-1)^{0.31} \quad \text{for } R \simeq 1.4 \tag{26}$$

$$q'' \propto R^{1/5} \quad \text{for } R \gg 1 \tag{27}$$

This is in limited agreement with the data obtained by Farhadieh and Baker[7] and in excellent agreement with the data obtained by Epstein and Grolmes.[10]

Appendix A: The Assumption of Thin Film Separating Pool and Substrate

It was assumed in the section on modeling that a thin film separates the pool and the substrate. This was done in spite of the fact that the melt and the pool are miscible liquids. In this appendix, it will be shown that fresh melt is generated at a faster rate than can possibly be diffused into the pool. Figure A1 depicts the physical model used here.

As soon as the pool comes into contact with the melting surface, the two transient processes of heat and mass transfer into the pool are initiated. The heat flux from the pool to the melting substrate may be written as

$$q'' = \frac{2}{\pi^{1/2}}\frac{k_p}{(4\alpha_p t)^{1/2}}(T_p - T_m) \tag{A1}$$

This heat flux causes a melt mass flux of

$$\dot{m}'' = \frac{q''}{h_{sf}} = \frac{2}{\pi^{1/2}}\frac{k_p/h_{sf}}{(4\alpha_p t)^{1/2}}(T_p - T_m) \tag{A2}$$

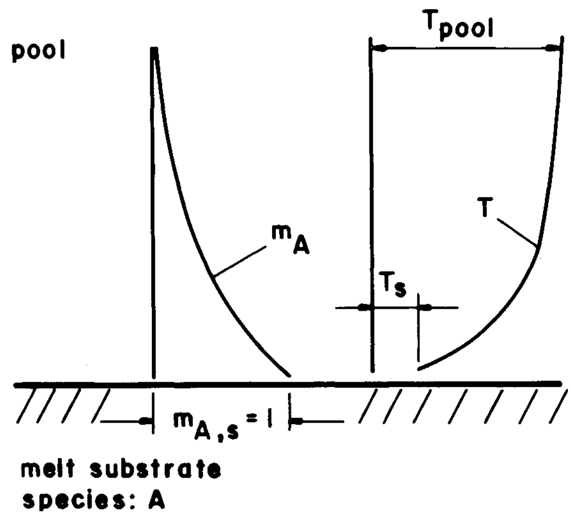

Fig. A1 Transient heat conduction and mass diffusion into the pool.

Meanwhile, the mass diffusion of species A (melt material) into the pool based on transient diffusion may be written as

$$\dot{m}''_D = \frac{2}{\pi^{1/2}} \frac{\rho_p D}{(4Dt)^{1/2}} (m_{A,s} - m_{A,p}) \tag{A3}$$

where $m_{A,s}$ and $m_{A,p}$ are the mass fraction of the melt material at the melt interface and in the pool. Assuming that $m_{A,s} = 1$ (pure melt at interface) and $m_{A,p} = 0$ (no melt material in the pool), Eqs. (A2) and (A3) may be combined to obtain

$$\frac{\dot{m}''}{\dot{m}''_D} = \left(\frac{\alpha_p}{D}\right)^{1/2} \frac{c_{p,p}}{c_p} Ja \tag{A4}$$

If $\dot{m}''/\dot{m}''_D < 1$, it means that the melt is diffused into the pool at a rate faster than it is produced. On the other hand, $\dot{m}''/\dot{m}''_D > 1$ means that there is an excess amount of fresh melt available at the interface and that it would provide for sustaining a thin layer of melt at the interface. Considering the combination of CH_2I_2 – Benzene studied by Catton et al.[8] and estimating that $D \sim 1 \times 10^{-9}$ m^2/s, the result is $\dot{m}''/\dot{m}''_D \simeq 1$ for low pool temperatures and $\dot{m}''/\dot{m}''_D \simeq 5$ for higher pool temperatures. Therefore, it seems reasonable to assume, for the range of parameters considered here, that a thin layer of melt separates the pool and the substrate. However, for those cases of $\dot{m}''/\dot{m}'''_D \leq 1$, an appropriate analysis and experimental investigation is recommended.

Appendix B: Assumptions of $\delta/\lambda \ll 1$ and $\eta(0)/\lambda \ll 1$

Assumptions 5 and 6 in the section on modeling were necessary in order to neglect melt layer inertia forces and assume a radial flow in the melt layer. In this appendix an attempt will be made to develop a criterion for the range of governing parameters, i.e., R and Ja/Pr, over which these assumptions are valid. The parameter δ/λ may be written from Eqs. (11), (2), (20), and (3) as

$$\frac{\delta}{\lambda} \simeq \frac{1}{6.6}\left[\frac{Ja}{(R+1)Pr}\right]^{1/5} \tag{B1}$$

To have $\delta/\lambda \ll 1$, (say $\delta/\lambda = 0.1$), one may obtain

$$\frac{Ja}{Pa} < \frac{8}{R+1} \tag{B2}$$

For a nominal value of $R = 2$, Eq. (B2) requires $Ja/Pr < 2.67$, which is quite satisfied for the range of our interest (see Fig. 2).

The parameter $\eta(0)/\lambda$ may be written from Eqs. (13), (3), (19), and (B1) as

$$\frac{\eta(0)}{\lambda} = \frac{6n}{3-n}[2\pi(\delta/\lambda)^2 - \ln(2\pi^{1/2}\delta/\lambda) - 1/2] \tag{B3}$$

For $\delta/\lambda < 0.1$ and $n = 1$ or 2, $\eta(0)/\lambda$ is calculated from Eq. (B3) to be always less than 0.1, and hence our assumption is justified.

References

[1]Taylor, G.I., "The Instability of Liquid Surfaces When Accelerated in a Direction Perpendicular to Their Planes, Part 1," *Proceedings of the Royal Society,* A-201, 1950, pp. 192–196.

[2]Bellman, R. and Pennington, R.H., "Effects of Surface Tension and Viscosity on Taylor Instability," *Quarterly of Applied Mathematics,* Vol. 12, 1954, pp. 151–162.

[3]Zuber, N., "On the Stability of Boiling Heat Transfer, *Transactions of the ASME,* Vol. 80, 1958, pp. 711–720.

[4]Berenson, P.J., "Film Boiling Heat Transfer from a Horizontal Surface," *ASME Journal of Heat Transfer,* Vol. 83, 1961, pp. 351–358.

[5]Dhir, V.K., Castle, J.N., and Catton, I., "Role of Taylor Instability on Sublimination of a Horizontal Slab of Dry Ice," *ASME Journal of Heat Transfer,* Vol. 99, 1977, pp. 411–418.

[6]Taghavi, K., Dhir, V.K., and Catton, I., "Thermal and Hydrodynamic Phenomena Associated with Melting of a Horizontal Substrate Placed Beneath a Heavier Immiscible Liquid," *ASME Journal of Heat Transfer,* Vol. 101, 1979, pp. 318–325.

[7]Farhadieh, R. and Baker, L. Jr., "Heat Transfer Phenomenology of a Hydrodynamically Unstable Melting System," *ASME Journal of Heat Transfer,* Vol. 100, 1978, pp. 305–310.

[8]Catton, I., Brinsfield, W.A., and Ghiaassiaan, S.M., "Heat Transfer from a Heated Pool to a Melting Miscible Substrate," *ASME Journal of Heat Transfer,* Vol. 105, 1983, pp. 447–453.

[9]Fang, L.J., Cheung, F.B., Pederson, D.R., and Linehan, J.H., "An Experimental Study of Natural Convection Melting of Ice in Salt Solutions," Paper 84-WA/HT-55, ASME Winter Annual Meeting, 1984.

[10]Epstein, M. and Grolmes, M.A., "Natural Convection Characteristics of Pool Penetration Into a Melting Miscible Substrate," *ASME Journal of Heat Transfer,* Vol. 108, 1986, pp. 190–197.

[11]Taghavi, K. and Dhir, V.K., "Taylor Instability in Boiling, Melting and Condensation or Evaporation," *International Journal of Heat and Mass Transfer,* Vol. 23, 1980, pp. 1433–1445.